PAR-003-1162002

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Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

August / September - 2020

CMT - 2002 : Mathematics

(Complex Analysis)

Faculty Code : 003

Subject Code : 1162002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures on the right indicates marks.

1 Answer Any **Seven** questions : **2×7=14**

(a) Write the statement of Cauchy's Integral Formula (1^{ST} Version).

(b) Define bounded variation and rectifiable path.

(c) Evaluate $\int_{\gamma} \frac{1}{z} dz$ where $\gamma(t) = e^{int}$ for $n \in \mathbb{N}$ and $t \in [0, 2\pi]$..

(d) Prove that $f = g$, if the set $\{z \in G : f(z) = g(z)\}$ has a limit point in G where $f, g : G \to \mathbb{C}$ is an analytic function and G is an open connected subsed of $\mathbb{C}$.

(e) Evaluate $\int_{|z|=3} \frac{f(z)'}{f(z)} dz$ if $f(z) = \frac{(z^2+1)^2}{(z^2+3z+2)^3}$.

(f) Write the statement of Taylors's Theorem.

(g) Define Meromorphic function and also write one example of Meromorphic Function.

(h) Show that $f : \mathbb{C} \to \mathbb{C}$ be defined as $f(z) = \sin z$ is not a bounded function.

(i) Define Pole and find the pole of $f(z) = \frac{e^{2z}}{z(z-1)}$ for $z \in \mathbb{C}$ also write the order of each pole.

(j) Write the statement of Open mapping Theorem.

2 Answer Any **Two** of the following : **7×2=14**

(a) Let G be an open connected subset of $\mathbb{C}$ and $f : G \to \mathbb{C}$ is an analytic on G with $f'(z) = 0$ for all $z \in G$ then show that f is constant function also show that one cannot drop the condition of connectedness in the above statement.

(b) Prove that $e^{z+w} = e^z \cdot e^w$ for all $z, w \in \mathbb{C}$ and $\overline{e^z} = e^{\overline{z}}$ for all $z \in \mathbb{C}$.

(c) Show that for an analytic function $f : G \to \mathbb{C}$ where G be an open connected subset of $\mathbb{C}$ and $G^* = \{\overline{z} : z \in G\}$ then $f^* : G^* \to \mathbb{C}$ defined by $f^*(z) = \overline{f(\overline{z})}$ for all $z \in G^*$ is analytic on G^*.

3 All are Compulsory. Attempt one pair of (a) and (b). **7×2=14**

(a) Prove that the representation of Bilinear Transformation is not unique.

(b) Show that every Bilinear Transformation $S \neq I$ has atmost two fixed points and deduce that if S and T are Bilinear Transformation such that $S(z_i) = T(z_i)$, for $i = 1, 2, 3$ for some distinct $z_1, z_2, z_3 \in \mathbb{C}_\infty$ then $S = T$.

OR

(a) Prove that if $f\,g = o$ then $f = 0$ or $g = 0$, provided f and g are analytic function on an open connected set G.

(b) Prove that $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ is any integer, where $\gamma : [0,1] \to \mathbb{C}$ be any closed rectifiable curve and $a \notin \{\gamma\}$.

4 Answer Any **Two** questions : **7×2=14**

(a) Let γ be a closed rectifiable curve in $\mathbb{C}$ then prove that $n(\gamma; a)$ is constant for "a" belonging to component of $G = \mathbb{C} - \{\gamma\}$. Also show that $n(\gamma; a) = 0$ for "a" belonging to the unbounded component of G.

(b) Write the statement of Cauchy Integral Formula for Derivatives (2nd Version) and show that $\int_\gamma \frac{1}{(z-a)^n}\,dz = 0$ for every integer $n \geq 2$, provided γ is any closed rectifiable curve in $\mathbb{C}$ and $a \notin \{\gamma\}$.

(c) State and prove Liouville's Theorem.

(d) Write the statement of Identity Theorem and show that there does not exists an analytic function $f : \mathbb{C} \to \mathbb{C}$ such that $f(z) = z$, for all z such that $|z| = 1$ and $f(z) = z^2$, for all z such that $|z| = 2$.

5 Answer Any **Two** of the following questions : **7×2=14**

(a) State and prove Cauchy's integral formula (1st version).

(b) Prove that if $f : G - \{a\} \to \mathbb{C}$ be an analytic function and α is pole of f then there exists $m \in \mathbb{N}$ and an analytic function $g : G \to \mathbb{C}$ such that $f(z) = \frac{g(z)}{(z-a)^m}$ for all $z \in G - \{a\}$.

(c) Let A_1 and A_2 be two circles in $\mathbb{C}_\infty$ and S be a Bilinear Transformation such that $S(A_1) = A_2$ then prove that if z^* *and* z are symmetric with respect to A_1 then $S(z^*)$ and $S(z)$ are symmetric with respect to A_2.

(d) Write the standard form of Bilinear Transformation and prove that every Bilinear Transformation preserves cross ratio.